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THE MATHEMATICS TEACHER

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TEACHING PUPILS THE CONSCIOUS USE OF A TECHNIQUE OF THINKING[1]

By ELSIE PARKER JOHNSON
Oak Park High School, Oak Park, Ill.

The primary aim of all education is to cultivate the individual's powers of thinking. John Dewey says "A being who could not think without training could never be trained to think; one may have to learn to think well, but not to think." The work of the teacher is to enable the pupil to employ more economically and effectively the powers which he already possesses. That these natural resources which must be used are widely varying in power and capacity, our modern methods of testing general intelligence have indubitably shown. Our hope is to foster the powers of each individual so that he can make the most of his original equipment. The problem before the instructor is how to accomplish this end most easily and thoroughly.

Dewey says further that "As the growth of the body is through the assimilation of food, so the growth of the mind is through the logical organization of subject matter." That the subject matter thus to be organized is not confined to one narrow field is his belief, for he asserts that "Any subject, from Greek to cooking, and from drawing to mathematics, is intellectual, if intellectual at all, not in its fixed inner structure, but in its function—in its power to start and direct significant inquiry and reflection. What geometry does for one, the manipulation of laboratory apparatus, the mastery of a musical composition, or the conduct of a business affair may do for another." This is, of course, equivalent to saying that the primary aim of education is not the imparting of all possible facts but rather to develop the attitudes of discrimination, suspended judgment and initiative and to train the powers of logical thinking. Indeed it is nothing short of saying that the disciplinary aims are of the greatest importance.

[1] Read before the National Council of Teachers of Mathematics, Chicago, Feb 28, 1924.

Up to a very recent date the validity of these disciplinary aims has been questioned because of the uncertainty of the transfer of training. However the subject is no longer one of controversy as the psychologists now agree that the effects of training do transfer from one field of learning to another. Miss Vevia Blair of the Horace Mann School has completed the digest of experiments on this subject which was begun by Dr. Rugg some years ago. The results of her investigations are given in the chapter on "The Present Status of 'Disciplinary Values' in Education" in the Report of the National Committee on Mathematical Requirements. After an analysis of the experiments along this line from the Thorndike-Woodworth experiment in 1901 down to the present time and a digest of the answers of twenty-four leading psychologists to questions concerning their stand on the matter she draws a number of conclusions of which I give those which bear directly on our discussion.

"(1) The two extreme views for and against disciplinary values practically no longer exist. As the question now stands, as transfer of training, the psychologists quoted here almost unanimously agree that transfer does exist.

(4) The amount of transfer in any case where transfer is admitted at all, is very largely dependent upon methods of teaching. This is probably the strongest note struck by the psychologists in their comments. Twelve of them out of the twenty-four make some explicit reference to the matter.

(5) A majority of the psychologists seem to believe that, with certain restrictions, transfer of training is a valid aim in teaching.

(6) Transfer is most evident with respect to general elements —ideas, attitudes and ideals. These act in many instances as the carriers in transfer. Often they form the common element held to be the sine quo non of transfer."

I have quoted these conclusions of Miss Blair in full because they justify the hope that training in the attitudes of suspended judgment, accuracy, orderliness, concentration, and initiative, the ability to analyze a complex situation and the ability to generalize can be taught to a pupil by means of any subject which permits a variety and change of ideas to be combined into a single trend moving toward a unified conclusion. But in geometry more than in any other subject taught in high school

it is easy to arrange situations in which the student can analyze and discover relationships without a multiplicity of irrelevant details. The factors used in arranging these situations in geometry, such as the triangle, line, angle, circle, etc. are the simplest ones possible, so that the mental processes are veiled comparatively little by the presence of extraneous factors.

But we must remember that neither the possibility of transfer nor the nature of the subject makes sure a certain kind of mental training for the pupil. The method of instruction has much to do with this. Dr. Judd has emphasized this point in these words "If ordinary school training does not transfer from one field of experience to another, this is not due to the inability of the human mind to transfer its training. The lack of transfer is in many cases due to the clumsy, stereotyped way in which ideas were put into the mind. The absence of general ideas and general habits of thought resulting from school work is due to poor teaching rather than to any limitation of the mind."

Our argument up to this point might be summed up as asserting: first, that our object as teachers is to train our pupils to think better; second, that the consensus of opinion at the present is that when a pupil has discovered the method of analyzing problems and of seeing relationships he will tend to treat other similar problems in the same way; third, that geometry furnishes a convenient field for this training; and fourth, that the transferable benefits received from the subject will depend largely on how the subject is taught. This then is the crux of the whole matter, as far as teachers of geometry are concerned—how can geometry be taught so as to impart the largest amount of transferable skill in thinking?

The traditional method of instruction has been to let the student discover for himself a method of reasoning which he thereafter uses without in many cases being aware of the fact that he is using that mode of procedure. Of late, teachers have endeavored to give the pupil some idea as to the technique employed and have taught the use of analysis. This is undoubtedly better than no technique at all, but it tends to become purely mechanical and formal as it ordinarily is used. Furthermore, this is not the complete representation of the mental processes through which one goes in solving a problem. A complete act of thought consists of more than one analysis, which is always

correct, and the ensuing synthesis. Thus only a part of the complete act of thought is pointed out to the pupil. John R. Clark suggested to the writer that pupils would be better able to solve geometry originals if they were taught to use consciously the steps given by John Dewey as constituting a complete act of thought. It seemed plausible that by consciously using all the steps in a complete act of thought, which were pointed out to him, the pupil would avoid the waste of time and the discouragement involved in discovering more or less thoroughly a method which he would probably use unconsciously. It also seemed possible that such a method would be more likely to carry over to other fields than would a less definite method of thinking of which the student was only partially aware.

The question to be answered by experimentation was then formulated as follows: Can pupils of geometry be taught to prove theorems more economically and effectively when trained to use consciously a technique of logical thinking; and furthermore does such training, more than the usual method, increase the pupil's ability to analyze and see relationships in other nongeometrical situations?

The experiment was first conducted with two classes in plane geometry for one school year of thirty-six weeks. The control group consisted of 25 members, while the experimental was 27 in number. The former group was selected as the control group because its median on the Chicago Group Intelligence Test was 51.5 as contrasted with the median of 46 for the other group. This meant that the bright group would be instructed in the ordinary way while the duller group would be used for the experiment. Then, surely, any superior ability to carry on logical thinking at the end of a year's instruction by the new method, if shown by this group, would indicate advantages in the method. It may be noted here that the average for the control group on this Chicago test was 51.3 while that of the experimental was 47.1 so that the averages could be used to express the relative abilities of the classes as well as the medians. It may also be noted that while the experimental group had one pupil whose score was the highest of all, it also had the lowest of all, and while it had three above 60, it had seven below 40 as contrasted with the two above 60 and the three below 40 of the control group. It is clear from these facts that the control group was

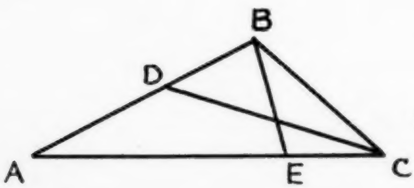
more uniform in ability while the experimental contained a wider range of capacities, another point to be considered a handicap in instruction.

Both classes recited for periods of 40 minutes five times a week, the control group at 8:45 A. M. and the experimental at 11:05 A. M. W. H. Winch in an article "Mental Adaptation During the School Day as Measured by Arithmetical Reasoning" (Part 2 Journal of Educational Psychology Vol. 4 p. 84) finds that correlations would indicate that the later hours of the morning are better for problematical arithmetic. But as this contradicts a previous experiment, and as the later period in the morning was often shortened on account of the assembly period which preceded it on Friday, the matter of the hour could not be considered as an advantage for the experimental group.

The aim of the instructor was to show to the experimental group how any problem in real life is solved by a definite train of mental processes which may be considered as a technical mode of procedure in solving a problem or in thinking through a difficulty. The example first used was that of the problem which the instructor herself had recently had to solve of opening the drawer of a dresser, when the varnish had become softened by the warm weather and the two surfaces were stuck together. A picture of the dresser was put on the black board and the information given that the article of furniture was a beautifully finished piece of modern construction. She then explained that the drawer refused to open and that she desired some assistance from the class in solving her problem. They then demanded more explicit details about the difficulty. They desired to know whether the drawer was locked, or hindered from opening by an article within it or whether the wood had swelled or the varnish melted. That is, they wanted the exact nature or location of the difficulty. This point having been settled the pupils began giving suggestions as to how the drawer might be opened. These suggestions were considered by the class and rejected as they were found to be unsuitable for some reason. For instance, the suggestion of running a thin knife between the two adhering surfaces was rejected because the fine finish of the wood would be marred by such treatment. The suggestion of pulling out the drawer above and then pushing out the drawer from within was not accepted because the fact that the dresser had

been described as of modern construction meant that the drawers were of dust-proof construction and hence this could not be done. Altogether ten different suggestions were offered, and it was actually the method offered as the tenth which had really been used to open the drawer. The successful use of this method was its verification.

From this simple example, so close to the pupils' experience that each one had prior knowledge and experience upon which to draw for suggestions, the instructor proceeded to show how any problem or difficulty is solved. Then the class and the teacher together solved the original exercise which is given below.



Given: $AC > AB$
$DB = EC$
To Prove: $DC > BE$

The class offered three possible methods of solution of which it finally discarded two and showed that the third gave the desired results. The teacher then compared the method of solving the difficulty with the dresser drawer and that used in solving the geometry original, making clear to the class the fact that the two modes of procedure were identical. After several such class discussions of practical, homely examples the analysis of a complete act of thought as given by Dewey in his "How We Think" was deduced and outlined as follows: (1) a felt difficulty; (2) its location and definition; (3) suggestion of possible solutions; (4) development by reasoning of the bearings of the suggestion; (5) further observation and experiment leading to its acceptance or rejection.

This method of discussing a practical problem of everyday life in class, receiving the suggestions offered by various individuals, these suggestions being followed by the reasoning of the class and rejected until one could be found which agreed with the facts given, served to arouse an interest in a problem-solving situation and then to make clear the steps in a complete act of reflective thinking. Such difficulties as how to select the material for a certain garment from several samples,

or how to get to the city when a street car strike is in progress were used to point out the steps in the solution of a problem by exclusion. The method of opening a drawer when a large box inside the drawer hindered its opening was a problem in which the class was keenly interested and which served admirably for the rehearsal of the steps in the act of problem-solving. These examples may seem absurd to one whose primary interest lies in the subject-matter of geometry, but these are familiar and comprehensible ideas to the immature student of geometry and may be used to bridge the gap between his everyday life and this study which is seemingly so different from anything in his experience. Pupils have hailed the originals with joy when they have seen the technique of attack, but I do not believe that this feeling of confidence and the same understanding of the steps in the act of thinking would have resulted had less familiar examples been used. Also, with material drawn from the usual sources of the illustrative and problem material, such as physics, the pupil would not have the wealth of prior experience which yields the material for the suggestions.

Out of the constant examination of acts of reflective thinking resulted, not only a familiarity with the steps in logical thinking, but also an actual interest in attacking problems. The pupils had seen that success in reaching a correct conclusion depends to a large extent on the calling up of related ideas and of many possible solutions. This tended, as can be shown by actual results, to prevent discouragement when the first solution attempted did not prove satisfactory. At two times after this training had been given to the experimental group, an original exercise was given to each group and a record kept of the time each individual worked. These tests were about six weeks apart, and were filed away without being corrected until after the class disbanded. No other such work was kept, so that these results are representatives of a random sampling of their work. The data on the first exercise given on February 19th shows that the members of the control group gave up trying sooner than the members of the experimental group, as twice as many quit before the end of the time allowed. On the original given on April 9th this is shown more strikingly, as three times as many of the control group as of the experimental group quit before the end of the time allowed.

Furthermore, acquaintance with the steps in thinking showed the pupils that the best thinker does not stay too long on one suggestion, but discards the erroneous ones quickly and then goes on to another. This is undoubtedly the reason that on the originals just mentioned, the control group found only two methods of proof while the experimental group found four methods. And probably this discarding of a suggestion which does not give the desired results and quickly trying another, as well as the conscious use of the other steps in the complete act of thought, explain why the number of correct or almost correct proofs was in one case one and a half and in the other two and a half times as great in the experimental group as in the control group. Undoubtedly, it would appear that the control group were purposely poorly taught or such differences would not appear, but such was not the case, as the enthusiasm of the teacher in trying the new method made the problem of teaching the subject much more interesting and the only difficulty was in restraining the impulse to let the control class share the ideas being tried out with the other class.

The standardized Silent Reading Test devised by Walter S. Monroe was given at three times during the experiment. This is a fairly good test of ability to reason along lines other than geometrical. On the test given prior to training the control group had a higher average than that of the experimental group but after training the experimental group had the higher average. Also the correlation between the scores on the Chicago Group Intelligence Test and the scores on the Reading Test improved more in the case of the experimental group.

An examination of the grades on the final examination on the subject discloses that the median for the control group was 70 and that of the experimental 75. Or, if one prefers to use the arithmetical mean, that for the control group was 71.52 and that for the experimental was 75.38. To appreciate the significance of these figures one must remember that the two classes were not even equal in ability, but that the median of the control group on the Chicago Group Intelligence Test, as well as the arithmetical mean, was higher than that of the experimental group.

In order to check these results, the experiment was repeated, although the exigencies of program making in a large school

limited the time of the experiment to one semester. As it is necessary for the pupil to acquire some knowledge of the terminology of the subject and of at least a few geometrical facts before being introduced to this phase of the work, the time left for instruction in the method and for training in its use was brief. Hence the results are probably not as conclusive as they would be at the end of an entire year's work.

This time the groups numbered 25 for the control and 26 for the experimental. The average *IQ* of the control was 109.8 and its median was 110, while the average *IQ* of the experimental was 107.7 and its median 108. These were found from the scores of the two classes on the Terman Group Test of Mental Ability which was administered at the beginning of the experiment. The range of *IQs* in the control group was from 90 to 127 and in the experimental from 89 to 126. Thus as in the previous experiment the training in the new method was given to the duller group. That this was the slower group can not be doubted when one notes that the median *IB* of the group as determined from the Otis Higher Examination was 121.5 as contrasted with 134 for the control group. The corresponding arithmetical means were 123.3 and 133. The classes were carefully balanced as to sex and age so that those factors could not affect the results.

The Burt Reasoning Test Form A as arranged by Miss Elizabeth Bruene in her *Master's Thesis 1920 The University of Chicago* was given to both classes before the training began. This is test compiled from a series of questions devised by C. Burt and has to do with material other than mathematical, being situations of ordinary life which require some reasoning in order to discover the answer. For instance one question is as follows: "If the train is late he will miss his appointment; if the train is not late he will miss the train. State whether he kept his appointment." Another example is: "Dismal Johnny said to Sunny Jim—'If I marry I shall be miserable, because I shall be bothered with looking after my wife; if I don't marry I shall still be miserable, because I shall have no wife to look after me. So in either case I shall be miserable.' Sunny Jim replied—'On the contrary, you ought to be happy in either case; for, if you do not marry, you will be happy, because you will not be bothered with looking after your wife, and——————.' How do you think he finished his argument?" On this test,

which obviously tests reasoning ability on subjects which have nothing to do with geometry, the control group had an arithmetical average of 8.1 and the mode was 9, while the experimental had an arithmetical mean of 8 and a mode of 7. After training as described in the previous experiment, the tables were turned, the control group having an arithmetical mean of slightly less than 8 with a mode of 8, and the experimental the arithmetical mean of 8.1 and the mode of 9.

Original theorems in geometry were given to the classes before and after training in the conscious use of a technique of thinking. While the classes had the same number of correct proofs on the first one, on the second one given after training the experimental group had one and one-third as many complete and perfect proofs as the control, while counting also the proofs which were on the right track but not complete the ratio was as 12 to 21 in favor of the experimental group. As in the previous experiment, it was found that those trained in the new method exhibited more perseverance in endeavoring to find a proof. In this case one and a half times as many of the control group quit before the end of the time allowed as in the experimental group. Also the control group found only one method of proving the original, while the experimental group found three methods of proof.

These differences may appear small when taken separately, but considering that they are, in both experiments and in every test, in favor of the experimental group after training, they seem very significant. In fact, these data would seem to offer conclusive evidence, in so far as one experiment can be considered to do so, that when pupils are taught to use, consciously, a technique of logical thinking, they try more varied methods of attack, reject erroneous suggestions more readily, and without becoming discouraged maintain an attitude of suspended judgment until the method has been shown to be correct. This results, as our data on the proofs of geometrical theorems show, in greater ability to prove geometry originals and a superior grasp of the subject. These results alone would justify a further trial of the method as offering a solution of the problem of directing study.

However the data on the reasoning tests would seem to indicate that such training in logical thinking with the materials of

geometry tends to carry over these methods of attack and these attitudes to other problem situations not concerned with geometry. If this be true, then indeed, our highest claims for the advisibility of teaching how to think by teaching the conscious use of the steps in a complete act of thought are realized—for in the words of the man upon whose analysis of thinking this experiment depends, "What is important is that the mind should be sensitive to problems and skilled in the methods of attack and solution."

HABIT IN THE EDUCATION PROCESS

By PROFESSOR W. PAUL WEBER,
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Sec. 1. *State of Mind.* Let us call any thing that enters into the complexity of a state of mind an *element of state of mind.* These elements are obtained through the sense experience, memory and higher mind processes. Thus any thing that comes into consciousness through these means is an element of state of mind.

Sec. 2. *One Primary function of mind* is its capacity to determine at any time what elements shall enter consciousness and receive attention. This implies what is ordinarily termed *voluntary control of attention.* Individuals vary greatly in this power. There is of course the non-voluntary aspect which is not considered in this discussion.

It is admitted that *proper exercise and instruction* will strengthen this control and direct it to useful purposes. This is *one of the objects* of education. A mind with this primary function well developed affords great possibilities. The acquisition of the proper kinds of elements to be stored in the memory will be an important consideration in this connection. This implies that some control of the sources from which the mind is furnished the elements during the developing period of life is necessary.

Sec. 3. *Two Fundamental Principles Will Now Be Stated.*

I. Every state of mind tends to arouse action. The action may be advantageous or disadvantageous. There is the possibility of neutral action or a sort of status quo attitude. This is action, however, within the meaning of the term.

II. The kind of action is determined by the state of mind immediately preceding it.

Sec. 4 *Implications and Comment.* From these principles it is evident that idleness is unnatural. Control of the conditions that furnish the elements that enter states of mind is desirable, even necessary. The elements planted in the developing mind should be such as would form a healthy basis of judging values of states of mind.

All this amounts to the formation of habits of sufficient strength to be trusted to direct the behavior of the individual. Thus, means of forming suitable habits becomes the great problem in the education of the individual.

Sec. 5. *Habit Defined.* When an act has been performed so often that the voluntary effort required to perform it has been noticeably reduced below that required for the first performance, habit is said to exist with reference to that act.

Under this definition it is evident that habits are of all degrees of strength from the merest probability of the habitual action to a considerable certainty of the action. This implies that habits that are considered important enough to be cultivated should be brought to a *sufficient degree of strength to function in life.*

Sec. 6. *Function of Habit.*

I. *Habits are essential to efficiency* in thinking and in body movement. This is universally admitted and implied in the training processes employed in the preparation of individuals for every walk of life, nearly, except unskilled labor. Intensive training is necessary to many activities.

II. *Habits limit the varity of choice of acting* in a given situation. For if an action that is habitual can be taken it is more likely to be taken than some new action, other conditions being equal.

Sec. 7. *Variety of Habits is necessary* to readiness and efficiency in a sufficient number of actions to satisfy the ordinary conditions of living. There is virtually no limit to the number of habits that might be considered for cultivating. Only a limited number of habits can be efficiently cultivated by the individual. This is due to the natural limitations of capacity and opportunity of the individual.

Sec. 8. *To secure habit* practice (drill or training) must be imposed. This is well recognized everywhere. It will be a problem for discussion later in this paper to point out certain habits and if possible to suggest ways and means of securing them in connection with the teaching of mathematics.

Sec. 9. *A choice of a set of habits must be made* from the maze of possible habits that admit of cultivation. There are

a number of well recognized habits that it is quite generally agreed would be desirable to cultivate. This list is increasing from day to day. Just what and how many habits should constitute the direct object of school instruction would at this time be difficult to determine. A sufficient number have been suggested as fundamental to engage our energies for a while. When we have learned definitely how to secure these it may be possible to decide upon others to be added. Example of desirable habits will be given in the sequel.

Sec. 10. *Two-Fold Object in Teaching a School Branch.* It is believed that there will be quite a general agreement on the assertion that the object of using any subject in school must be: (1) *To furnish a certain amount of information* that it is expected will be useful in after life, (2) *to fix* or at least aid substantially in fixing *certain habits* that are of high value and of broad application in after life. Here arises a problem, viz., *To determine what desirable habits can be most easily and effectively fixed by any given branch. Further it must be determined how to present the subject to accomplish most effectively the objects it is intended to secure.*

Sec. 11. *Remark.* It is easier to teach a subject with the object of knowing the subject itself than to realize other educational values of the subject. It is probably true that some of the other values may accrue from the study as by product without any particular care. It is now quite generally recognized in education that such a method falls far short of full realization of the educational possibilities of the subject.

Sec. 12. *Some Principles to be Observed.* At the beginning of the course of instruction in any branch a decision should be made with regard to:

(1) *The topics to be included and the degree of intensity of the treatment from the informational point of view.*

(2) *The definite habits that can be cultivated by use of the subject matter employed.*

(3) *The contacts of the subject with other subjects that are to be pointed out and made clear during the course.*

To be sure these questions may not be easy to settle at first effort. With reference to the first of these points the two fun-

damental working principles laid down by the National Committee of the Mathematics Association of America on mathematical requirements should be carefully considered as a basis. These principles are the result of mature thought on the subject.

In regard to the second and third of the above principles there is much to be done before a definite statement can be made. It is here that research needs to be done with the view to arriving at some definite program.

After all this, there remains the problem of the best means of attaining the desired ends. Here again research must be done before we can proceed with confidence and certainty.

Sec. 13. *Transfer of Training.* It is no longer necessary in well informed circles to defend the principle of transfer. It is accepted as a working fact. But, be it understood that transfer depends upon several things, such as:

(1) The selection of the objects of instruction. That is, whether a subject is taught for itself alone or as an educational instrument.

(2) The method of presentation to the pupils.

(3) The generalized habits and outside contacts that can be realized by the instruction.

It may be regarded as settled that unless the conditions favorable to transfer are present in the instruction the transfer will be unsatisfactory.

Sec. 14. *Some conditions favoring transfer* may be mentioned by way of suggestion. (1) Develop the subject in a fashion somewhat similar to that actually experienced by the race or after the manner of investigators.

Of course this can be carried out only to a limited extent and in a simple way with young pupils. The heuristic method is here suggested. The object is to cultivate the habit of investigation to some extent in fields that come within the pupil's power.

(2) By examples show how the subject and its method may be extended to other fields. This is commonly styled applications.

This will lead to generalization, the most potent of the instruments of transfer. The habits of seeking applications and of generalizing are highly valuable.

(3) The steps in the development of the subject should be kept in full view of the pupils as far as possible.

Here can be cultivated the habit of seeking the modes of procedure in building up a body of thought. Even simple examples of scientific procedure carried out by pupils under guidance will be useful. This means study of material, organization and presentation.

(4) Pupils should be led as far as possible to make the transfers the subject admits.

There must be conscious effort and definite material employed to ensure this value. This connects directly with (2) above.

Sec. 15. *Comment.* It is to be understood that such subject matter as is to be taught will be taught thoroughly, be it much or not so much. A modest amount of subject matter, carefully selected and taught under the guidance of the above principles will be of far greater value to any student than a larger amount given in the traditional way. The recommendations of the National Committee may be profitably consulted regarding the subject matter to be included in the curriculum.

Some Generalized Habits and Attitudes That Have Been Shown to be Transferable

Sec. 16. *Attitude or Habit of Verifying.* Asking for the evidence instead of reasoning alone from assumptions and first principles.

To attain this, pupils should be taught to seek external evidence of the correctness of conclusions arrived at through reasoning. They are to learn to verify the results of their reasoning, as far as possible, by observation and experience with actual things. This habit can be had only by practice. The necessary practice must attend the presentation of the course.

Sec. 17. *The Problem Attitude or Vital Attitude.* Pupils should be trained to approach new situations as problems to be attacked and solved and not as something on which authority is to be accepted alone. Teach the pupil to study the situation by analyzing it and then by various trials to establish a chain of steps that will amount to a solution. Concrete problems furnish

material for part of this training. Original problems taken from the environment or from the pupil's individual experience should be added.

Sec. 18. *Attitude of Satisfaction with Accomplishment.* There is not enough respect paid by young people today to accomplishment as a virtue in itself. Any worthy piece of work well performed should be regarded as a creditable accomplishment and as deserving of some praise or recognition. The many more or less humble things done every day keep the world going. If these are well done the fact should be a source of satisfaction to the worker and should be credited to him by others.

Sec. 19. *Attitude of Correlating Theory with Applications.* This attitude is opposed to formalism and mechanical drill as an end. This correlation should favor keeping in touch with affairs of daily life. It is usually much easier to teach the formal phase of a subject than to teach the correlation phase. It is the latter that will be the source of satisfaction and of efficiency for the pupil in later life fully as much as the former. This is to be done by means of properly chosen problems from as many different fields as can be brought within the pupil's understanding.

Sec. 20. *Remark.* It is evident that to train pupils in the above habits the teacher much have considerable experience with things not immediately mathematical. The regular requirement of having pupils bring mathematical situations from their daily experience or from home experiences and from observing new buildings, bridges and all such things, as sources of problems will be valuable.

The above four general habits look toward the practical applications of the theoretical principles chiefly. There are other types of general habits. We mention the following:

Sec. 21. *The habit of accepting patient effort as the only reliable and honorable means* of realizing one's ambition in life.

The moral value of this habit can hardly be overestimated. Insistence upon application to one's work in school will aid.

Sec. 22. *The Habit of Accuracy in All Work.* No piece of work should receive full approval until it is as free from errors as it is possible, under the conditions.

Sec. 23. *The Habit of Caution.* This habit can be cultivated in mathematics. Occasional problems and questions that lead to inconsistent results and problems so stated that careless reading will lead to a result that is absurd may be suggested here, problems with little surprises, also.

Sec. 24. In close relation to the last may be mentioned the *habit of self reliance.* This can be cultivated in connection with a wide selection of material. Some suitably constructed simple catch questions and questions having ambiguous conditions may be used. Only simple cases within the understanding of the pupils should be used.

Sec. 25. *Habit of Sustained Attention.* It is desirable to cultivate the span of attention for all pupils. Theorems in geometry and certain types of problems such as originals in geometry and in algebra may have value in this training.

Sec. 26. *In General, Methods of Attack Are Transferable.* This has long been established. It follows that any general methods of procedure in problematic situations are of use in more than one field. This suggests the importance of emphasizing certain types of methods of attacking problems in the course of conducting the class work.

It might be possible to classify methods of attack and select the most important or the most feasible at any given stage of the instruction.

Sec. 27. *Remark.* It is not difficult to see that efforts to establish general habits such as are listed above is equivalent to an effort to establish the habit of *scientific method.* When any branch can afford an opportunity to do this in any considerable degree and at the same time impart a respectable modicum of usable information in subject matter, great educational possibilities are at hand. A responsibility in this regard rests upon the teacher and his superior officers. Textbooks could be improved in this respect.

A LABORATORY METHOD OF TEACHING MATHEMATICS IN THE CLASSROOM[1]

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When the term laboratory is mentioned in connection with any subject a mental picture is immediately formed of a classroom equipped with long and heavy tables, and numerous cases whose shelves are plentifully stocked with apparatus and equipment. Another seemingly indispensable addition also thought of is a manual or set of directions to guide the student in using the foregoing equipment to demonstrate the laws or principles under consideration at the moment. Although a laboratory is defined as a work shop devoted to experiments in any science for the purpose of observing its laws in operation, formulating its principles, and systematizing them, the question might well be raised whether it is essential that the student follow directions in performing an experiment or a series of experiments, and whether or not it is possible to do laboratory work in schools where classes cannot be equipped like laboratories. If it is, the great advantages derived from this method should be made available to all pupils and schools.

Leading educators realizing the need of such adaptation, after a careful study of the purpose of the laboratory method have formulated the following: "Laboratory methods provide the subject matter of instruction in the form of real, present experiences." This practice contrasts with the methods in which the subject matter was derived through the medium of books or teachers or from the past real experiences of students. Now, in the laboratory, real experiences are provided primarily for three purposes; namely (1) for information secured through observations, (2) for the experimental solution of problems guided by reflective thinking and (3) for the acquisition of skill in manipulation.

Securing information through observation is predominant in study in general. Thus we study the structure of plants and animals in biological laboratories. It is also predominant in many processes in chemical laboratories—for example in learn-

[1] Address delivered at Mathematics Council at Chicago February 23, 1924.

ing what kinds of precipitates are formed with various solutions of salts and acids. In mathematics this type of experience is derived from the intuitive method, a method of learning by looking. That is in the case of the three interior angles of a triangle. Here the student strikes off the angles of the triangle and when he places them together he observes that they form a straight line.

The reflective solution of problems by experimentation is present in the study of problems in physics (for example, in work with pulleys) and in chemistry (for example, in qualitative analysis of unknown compounds). The student in physics in working with pulleys, if he is doing reflective thinking, comes to the conclusion that, neglecting friction, the force necessary to lift an object is equal to the weight of the object divided by the number of strands in the pulley. Likewise in the qualitative analysis of compounds, the student must stop to think what happened when he was experimenting with the metals of the first group, the second group, etc. He must also reflect as to the results obtained when various solutions were combined in the experimental determination of the various metals. Mathematics has always claimed as one of the principal values of the subject that there are numerous opportunities for reflective thinking. In the solution of prose problems in algebra, and in the development of theorems in geometry the student, even in the complexities of a great body of formal work is required to exert a great mental effort which requires the highest type of reflective thinking.

Acquiring skill in manipulation is prominent in the biological sciences in dissection, and in chemistry in the construction of apparatus and the management of such processes as precipitation, drying, weighing, distillation, etc.

In mathematics the pupil develops technical skill in the use of his personal equipment, the ruler, compass, protractor and triangle, and in the construction of models, as rectangular, solids, cubes, pyramids, etc., when the need for same arises. He also acquires skill in manipulation by using these same instruments (many times as large as those of his personal equipment) at the blackboard. Mental skill of manipulation is acquired through the working of problems or exercises.

Danger of over emphasis of unessential materials.

The subject matter of laboratory instruction should be adapted to the social needs of the various classes of students to be found in high schools, and would thus necessitate relating the experimentation very definitely to processes that play a large part in the practical affairs of ordinary people. There is a danger in using a superabundance of material in experimentation instead of relating experimentation to a few large topics or problems. Pupils develop poor habits of experimentation when there are too many experiments and neglect the thought aspect of the work. Thus the laboratory activity becomes a mere matter of hurried manipulation. Again a great deal of time is frequently lost in going through experimentation and manipulation when the same real experience could be given more economically and effectively by having the teacher or student give the same demonstration before the class. The laboratory method might also be abused by being used indiscriminately, for example where reflective thinking and observation is more important than acquiring skill in manipulation. Thus highly specialized forms of motor skill should be avoided. Some have advocated laboratory methods in geometry, and emphasize the ability to turn out plates which requires some skill in mechanical drawing. This is unnecessary, as a student's understanding may greatly exceed his skill in drawing. If he can present the material neatly and clearly why bother about a highly specialized mechanical drawing.

One's mental attitude does not necessarily change just because he engages in certain physical manipulations and handles certain tools and materials. A student may acquire laboratory methods as so much isolated material, just as he may so acquire subject matter from a textbook. The problem of turning laboratory technique into intellectual account is infinitely more important than the utilization of information obtained from textbooks. Teachers know this, and though they are again and again impressed with the inadequacy of book instruction, ease their consciences by merely putting pupils through some form of laboratory exercises. To sum up this discussion, it might be stated that the most important aspect of the procedure is the extent to which it contributes to reflective thinking on the part of the student.

Having taken note of some of the dangers to the laboratory method, let us now turn to a discussion of some of its factors that led to its sucessful use.

(1) *Individual Differences.* The fact that children differ from one another is certainly obvious to any and all observers. These differences are pointed out not only with pride by parents when discussing their offspring, but an abundance of statistical material measuring range of ability is available.

The system of uniform instruction in classes tends to disregard these differences instead of taking them into consideration, because children differ among themselves. Even in a well-graded class though the majority of pupils are of nearly equal ability, some are always brighter and some are slower. They differ in temperament, some being students, others strongly emotional and still others practical. They also differ in their types of imagery, some being predominently visualisers, other motilcs and still others audiles. Again girls differ from boys in their characteristics. Pupils also differ in industry. Then again some pupils are impulsive and some are deliberate. How frequently does it happen that a teacher irritates the former in the old recitation method of teaching by forever checking their natural tendency to jump at conclusions; or the latter by urging them on to what seem to be impossibly hasty decisions. And how often does the teacher too vigorously oppose their natural bent and thus confuse and make the one sulky and the other nervous and tearful, and thus forever destroy any interest that the child might have in the subject.

The teacher if conscientiously attending to duty has to make a careful study of the classes to see which of the instincts are functioning at their maximum and upon that basis arrange his course.

2. *Pupils Learn by Doing.* For centuries chief reliance was placed upon curiosity and imitation. From the time of Froebel more use has been made of play, and lately constructiveness and the social instincts have begun to be utilized. Now it is interesting to note that this change of emphasis from one to the other is that the use of play and constructiveness does not cause teachers to discard curiosity and imitation. For when a new method is utilized, and even sometimes carried to excess, the result is that in the end it does not supplant the older valuable methods

but takes the place beside them if it demonstrates its value. Until but a few years ago children have been assumed to be merely intellectual beings and little stress has been laid upon their interest in constructing things.

In the majority of school rooms, practically the only educational activity that goes on is that of listening, absorbing, a state of passivity on the part of the pupil with the teacher as the main cog in the machine. It means the dependence of one mind upon another. The laboratory plan does away with this situation entirely. There has been a realization of the fact that pupils are exceedingly interested in muscular activity and since this involves work with concrete materials it was recognized that they could think better in terms of concrete cases which they could understand rather than memorize abstract material that is foreign to them. But we see that most of the constructive work that has been attempted in most places has been used as a center in the lower grades. It has been isolated in the higher grades from all activity and has been used but as illustrative material.

3. *Objectives of Teaching.* The common tendency has long been to emphasize the course of study or the prescribed requirements at the expense of the pupil's development and his real interests. The textbooks and any other information should be a means to an end and not an end in themselves for much of the information acquired is sooner or later lost.

Professor Henry C. Morrison[1] of the University of Chicago maintains that if a person has actually learned a thing he is a different person. Some people have the idea that education is the loading up of the mind with a stock of miscellaneous material. The mind however, is not a mere receptacle even though much of our past practice has been based on that assumption. It is rather something to be developed through information and many varieties of experience to modify the performance of the individual. It is the organ through which adjustment to environment is made. The adjustment is the objective of teaching. Some progress has been made in breaking down this mental receptacle theory, but there still remains plenty of its influence. Its down fall will come thru the scientific study of educational problems.

[1] The Teaching Technique of the Secondary School (class notes).

In the many mental and intelligence tests, we see the desire for results or the power to do, use, or understand something. This is more than the ability merely to reproduce a body of information. In the project method of teaching we want the pupil to experience certain things and form certain understandings and appreciations. By having him actively take part in a campaign for improving the appearance of his own neighborhood he is developing what is best and most desirable. Incidentally he is acquiring some valuable information even though he is not so conscious of this process now. The important thing is getting results thru him in his better understanding of living problems and his right appreciation of them.

Further evidence of this progress is observed in the attention of teachers to the study habits of pupils. They realize that more contact during the lesson learning process will influence the learning process advantageously, develop better habits of study and will also give better opportunity to modify character. During the supervised study period the teacher has a chance to become acquainted with the pupil's strong and weak points and also with his method of working. The instructor should not be concerned entirely that the pupils are getting all the information possible from what they are studying but rather how they are getting it and how it is affecting their conduct toward others and their appreciation of every-day affairs.

In our attempts to build up character and change performance we might think of these transformations in terms of attitudes or powers. The attitude can be thought of as consisting of an understanding and appreciation. It is not sufficient that an individual has only an understanding of many things. For example one may understand that it is dangerous to use the public drinking cup, but unless we have a thorough appreciation of infection we may take the risk. In the study of sciences we get into the scientific attitude of mind. We learn to weigh things carefully before deciding. We learn to control various factors of an experiment to improve our results, and also learn to isolate and vary one factor and keep the others constant. After we have studied the atomic theory we have different conceptions of a piece of iron, a concrete block, the air and all matter. Our entire attitude toward the physical universe has been changed and has become a part of us so that we can never again think

as we did in our former way. We are changed individuals as was previously pointed out. How frequently is history taught as an information subject. Instead of presenting the subject so that the learner can grasp the big movements intelligently, it is obscured by a maze of details such as the memory of dates, names, wars, campaigns, etc.

Some may object to this viewpoint and maintain that it is very difficult to secure new attitudes or powers in some people, for we must admit that people differ in the degree of adaptability, plasticity, or docility as we may wish to term it, but nevertheless that does not invalidate the idea that it should be the goal for the attainment of the average individual. We find at the present time that standard tests are replacing the old type of test and that they are destined to measure not mere information but changed attitudes, powers or adaptations, and we might say that the real test of a person's education is his voluntary conduct and power to act properly in life situations.

Professor Morrison has also found from his studies that pupils are apt to fall into three classes as far as learning is concerned. The first type he calls the lesson learner. This pupil is able to "get by," and adjusts himself beautifully to any school duties and assignments but later on in life even though he seemed to be a brilliant student his attainments are mediocre or else he has failed entirely. This procedure of lesson assignment and lesson learning is based on the wrong assumption that the pupil will make the proper transfer or adaptation. But in reality it is based largely on memory of facts and has not really become a part of the pupil's workable tools.

The next type he calls the transfer learner who can learn from the daily lesson assignments and usual order of procedure. He adjusts himself, however, with difficulty. He can use his information and gets along fairly well in outside situations, but the question remains whether the school is doing all it can for him. The answer is no.

The last type is the direct learner. The direct learner is one who is unable to get much from text books or lesson assignments. He is frequently of the radical type who never adjusts himself to the lesson-learning process, and is classed by the teacher as a mental defective and is usually headed for failure. He is the one who merits our consideration. A good illustration of this

third type was a certain inventor of high grade electrical machinery who was unable to learn his algebra and physics in school. He evidently lived in such a world of reality and activity as Professor Morrison puts it, that mathematical abstractions had no content or appeal to him. The strange thing about it is that later he acquired all the advanced mathematics he needed in connection with his work. His inventive stimulus motivated him. In fact he was always showing his inventive skill. But this type cannot be trusted to see the point in a lesson. It only confuses him. And here is the very justification of the laboratory method of teaching mathematics which I am now about to describe. That is, abandon the lesson hearing and lesson learning theory of teaching and substitute the direct method. Before giving the method, however, I shall take the liberty of describing in detail the equipment that is used in conducting this type of class room exercise.

Although it was mentioned in the beginning of this paper that laboratory tables and elaborate pieces of apparatus were unnecessary in the laboratory procedure, the writer does not wish to convey the impression that no equipment is needed. In each room the teacher has for his use or for that of the class, two large wooden protractors, two 30°, 60°, 90°, wooden triangles, and two 45°, 45°, 90°, triangles, six wooden straight edges with handles like those on a plasterer's boards, one dozen black board compasses, 1 box of 12 inch rulers, 1 box of lead pencils, several small brass protractors, and one box of lead pencil compass holders. In each room there is a cork bulletin board upon which the pupil's written work might be exhibited. In one room there is a five foot slide rule and a glass doored case for the storing of mechanical devices and models used in instruction or the result of the pupils constructive efforts. Each room has a case which has three shelves used for the note books of pupils of the first year of the Junior High School. In addition to the foregoing, each room has a section of squared blackboard for graphic problems and a supply of colored crayons. Scissors and string are also available.

Besides the equipment common to each room there is a full set of surveying instruments consisting of two transits, level, leveling rod, steel chain, several steel tapes, wire pins, and red and white sight rods. There are on hand several Duplex 10″

slide rules for use by either teachers or pupils, and we find that a great number of the pupils are getting rules for their personal use.

In addition to the general room equipment the individual pupil carries a large note book containing unruled paper 8′ x 11′, squared paper, several sheets of tracing paper, and attached to the rings of the note book is a ruler, a brass protractor and a pencil compass. If a pupil is working with theorems where demonstrative proofs are required he is furnished with a printed form known as "theorem paper". This adds to the neatness of his work and saves time in doing it.

The presence of the above equipment stimulates the pupil to make various models to illustrate some of the theorems in both plane and solid geometry.

After the foregoing consideration of the laboratory plan it is time to give the method of procedure. First of all the subject matter must be such that it can be used in the method to be described. For this purpose material is arranged in pedagogical units rather than in logical units. As explained by Mr. Breslich[1] in his paper in "The Unitary Organization of Mathematics," that subject matter is put together which is most economically and effectively learned together. It must be possible to present the unit as a whole in concise form so that the learner might have a clear and general conception of it. It must also be a compact body of closely related facts and principles susceptable to being mastered. Thus the first step in the laboratory procedure would be the presentation. Here the teacher presents in a short period of time, 10 to 20 minutes, the unit as a whole. He brings out the important facts and their relations to each other, and to the unit. This is really a concise expository survey to enable the class to see the road it is to travel, getting a complete though superficial notion of the unit itself. Thus at the end of the presentation the pupil has a definite idea of the unit, but needs to study further and in detail the various facts brought out by the presentation. A good illustration of the presentation of a unit would be the one which is concerned with the measurement of angles in terms of arcs of a circle. The teacher explains the meaning, purpose and value of

[1] Breslich, E. R. The Unitary Organization of the Mathematics of the Seventh, Eighth, and Ninth Grades. The Mathematics Teacher. April, 1923, Vol. XVI, No. 4.

this method of measuring, and shows the various positions of the sides of the angle with reference to the circle. He illustrates by means of drawings that the vertex of the angle may be within the circle, in the circle, or outside of the circle. He shows that in the first case that it may fall on the center or on some other point. In the second case, both sides may be chords, or one side may be a tangent and the other a secant. The theorems corresponding to the various cases are formulated. Thus the student learns a series of theorems which he recognizes as related to each other instead of a number of isolated theorems.

There are some who will object to this illustration and maintain that this is the one grand example of a unit in mathematics. To answer this objection, the following might be given as another case of a mathematical unit:

Unit—The Properties of Angles:

(1) Angles are seen in the class room, in models and in drawings. They occur in problems in surveying and navigation. An angle is formed by the rotation of a line.

(2) Angles are measured with the protractor. The degree is used as the unit of angle. Angles are classified.

(3) Angles of given sizes are drawn with the protractor. It is used to draw parallel lines, perpendicular lines, bisectors, triangles from given parts. This leads to some discussion of congruent figures and the establishment of basic theorems.

(4) Properties of angles studied by indirect measurement. Using methods as congruent triangle, scale drawing, similar triangle tangent ratio.

(5) Measuring angles leads to many functional relations as $x + y = 90$. $ax + bx + cx = 180$. Many arithmetic applications are given.

(6) Approximate measurements. Here, as in the case of line segments, the pupil appreciates the fact that measurement at best is only approximate.

The second step in the process is the assimilation period. The pupil now turns to the text book and reads with the purpose of being able to restate in his own words what he has read. A brief report is made by a pupil at the end of each section read, with an opportunity for asking questions and having a class discussion. Problems are then taken up. This is the time for the laboratory work or what is known at our school as the super-

vised study period. It is during this period that the teacher has an opportunity to observe the study habits of his pupils. This is done with the aid of a profile sheet.

Now let us see what the pupil is doing during this period. He may have come into contact with the isosceles triangle for the first time. In the ordinary type of teaching the triangle is defined and he is told to prove that if two angles of a triangle are equal the sides opposite them are equal and vice-versa. By the laboratory method the procedure would be somewhat as follows. The student draws a triangle having two angles equal and measures the sides opposite them. These being found equal the pupil now can see that if two angles of a triangle are equal the sides opposite them are equal, for he has had a concrete demonstration of the same. He should next draw a triangle having two angles equal to 60° and by measurement would discover that the triangle is equilateral. In the same manner with his protractor and straight edge he could construct a right triangle having its acute angles equal respectively to 30° and 60°. By measuring the sides opposite the 30° and 90° angles he could see that the hypotenuse is twice the shortest side. The demonstrative proofs of course would come later on in the mathematics course.

In another class a pupil might be learning to multiply $a + 7$ by $a + 4$. He does this by drawing a rectangle whose width is $a + 4$, and whose length is $a + 7$. He then divides this into four parts to show that one part is a^2; a second part is $4a$; a third part is $7a$; and the fourth part is 28. Thus he has expressed the area geometrically and algebraically. In another class a pupil may have drawn a right-angled triangle so that the hypotenuse is twice as long as one of the sides about the right angle. He may have lettered the hypotenuse $2x$ and the one side x, and he may have expressed the fact that the sine of angle $A = \frac{x}{2x} = \frac{1}{2}$. By measuring the angle opposite the short side or by recalling that it is equal to 30° he is enabled to find that the sine of $30° = \frac{1}{2}$. Many other illustrations might be given if time permitted.

In all of this work the student uses plain No. 6 notebook paper. Ruled paper is objectionable because the lines on the paper interfere with the lines in geometrical drawings. On

each paper the name of the pupil and the date is given in an assigned place, *e. g.*, in the upper right-hand corner. The name of the class—for example, Math. 1—and the page or the pages of the book is written in the upper left-hand corner. Parallel to the left edge of the page and about one inch from it, a line is drawn to leave a margin for marking and criticism by the teacher. The pupils write nothing on this margin. The perforations should always be on the left side of the paper, and pages should never be torn from the notebook. Only one side of the paper should be used and the number of the problem corresponding to the number in the textbook should be written near the left-hand margin. All work should be written with pencil (medium hard, never soft) because few pupils are able to make good drawings in ink. Work should not be crowded on the page. All drawings should be made with ruler and compass and all answers should be marked plainly.

In algebra the page should be divided into two parts by a vertical line. All clean work is then written on the left side of the line of division, all computations to the right in the proper horizontal position. The teacher can often get a better understanding of the pupil's difficulty from a study of his computations than from the "clean work." Knowing that the teacher may examine his computations, the pupil tries to work his problem correctly at the first attempt and becomes careful and exact in his work.

From the standpoint of mathematics it is essential that the written work should always be clean cut and neat. A careless piece of work, is frequently the cause of errors and often results in failure to work the problems. It is thus important that pupils be trained from the beginning to do neat work.

The use of scratch paper is prohibited, because it is probably one of the greatest causes contributing to careless written work. An easy cure for careless work is to allow the pupil to complete a given assignment and then require that it be rewritten in desirable form. During this period the teacher is the manager who is always on the alert and is amongst the pupils giving a direction here, a suggestion there in a low voice. The teacher does as little talking as possible, making only such comments that are called for and are vital, significant and helpful. It

might be mentioned here that all work is done in school and that we have no such a thing as homework.

After the assimilation period, which lasts from two to four weeks, depending on the length of the unit, pupils are then required to write an organization of the unit. This is somewhat like the outline that an author makes when he is writing a book. This is then followed by a recitation. Here each pupil is assigned a topic for a floor talk which he presents much in the same way that a teacher would present material when teaching a class. These topics may be on applications and uses of the unit; summaries of certain types of work, as outlines of various kinds of equations studied; methods of solving verbal problems; graphical methods; discussion of the nature of the roots of equations; number systems, etc.

The question might arise at this point as to how individual differences are provided for in this scheme. For each unit the minimum essentials have been determined and each pupil is required to complete them before taking up the next unit. Some pupils who finish ahead of time are given additional exercises to complete or are sent to the library to engage in some profitable reading. The markedly superior pupils are given supplementary projects to work out. The mathematics department of the University High School has worked up a list of supplementary projects suitable for the pupils' uses. Books containing the project materials are on the shelves of the mathematics section of the library and are always available. These projects consist of mathematical material or material related to mathematics. It might be stated here that some of these papers are as good as those frequently turned in by graduate students.

The next step in the procedure is the written test. This is always a departmental test. A key for scoring is made before the test is given. This key is then followed in the scoring of papers. It aims to eliminate the personality of teacher and pupil and thus makes the scoring uniform. Now it may be that some pupil has not been paying attention or that some points brought out have not registered with a certain pupil. The result is that he makes a poor showing in the test. The procedure here is to reteach the pupils until mastery takes place and the pupil can write a satisfactory paper. The teacher should always study the problem carefully and survey the ground or else many re-

teachings will be necessary. It may in some cases be necessary to call one or more of the other teachers into consultation and the supervisory help of the principal's office before a fault hitherto unnoticed by the teacher may be discovered.

In conclusion the values of the laboratory method may be summarized as follows:

1. The teacher has an unusual opportunity to develop the study habits of the pupil.
2. The failures are eliminated in that complete failure is unknown.
3. The pupil's power to retain what he has learned has been greatly increased.
4. It produces independent workers and thinkers.
5. It is an effective method of bringing home to the pupil the importance, meaning and appreciation of the subject material.
6. It brings about appreciation of the relationships of facts and principles to each other.
7. It supplies a motive for the study of mathematics and arouses interest in the pupil for the subject.
8. It develops good study habits.
9. It will add to the ability of the pupil to master the subject material with the least possible expenditure of time and effort.
10. It will do much toward the elimination of the most serious criticisms against mathematics, and it will make the position of mathematics in the curriculum more secure.

THE ORIGIN AND DEVELOPMENT OF OUR PRESENT METHOD OF EXTRACTING THE SQUARE AND CUBE ROOTS OF NUMBERS[1]

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I.

Our present method of extracting the square and cube roots of numbers by the orderly evolving of digits (Evolution) is based on two considerations: (1) the inversion of the binomial formulas $(a+b)^2 = a^2 + 2ab + b^2$ and $(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$; and (2) the place value of the digits in the Hindu-Arabic system of notation. The two principal devices for making an effective technique of evolution are: (1) the separation of the radicand into periods of two or three figures each; and (2) the employment of a methodical *schema* or setting-up of the work so as to facilitate the operations and show the stage of progress. With such comparative ease and sureness is this done by even an eighth grade pupil of today that it is hard for us to realize the slow and painful path that led to the world's mastery of this process.

For this same process did not come in a cloth from heaven as a ready-made gift to the children of men. It took a thousand years from Arybhata to Cardan to perfect this method, and two hundred years more to demonstrate its effectiveness when linked to the notation of decimal fractions advocated by Stevin. It is hoped that a knowledge of the history of this method will make the teacher sympathetic with and appreciative of the human efforts expended on this process, so that it may mean something more to him than a mere mechanical, result-producing device.

Consider, by way of contrast, the processes used by two of the best mathematicians of ancient times, Archimedes (c 212 *B. C.*) and Heron of Alexandria (*c* 200 *A. D.*). In Heron's *Metrika,* recently discovered in the Seraglio library in Constantinople, the Alexandrian surveyor derives the square root of 720 by taking the root of 729, the nearest square, which is 27; dividing this into 720 gives $26\frac{2}{3}$; averaging 27 and $26\frac{2}{3}$ gives $26\frac{5}{6}$ as an approximate root. (Method of Averages). To find the cube

[1] Paper read before the Ohio Section of the Mathematical Association of America, at Columbus, March 30, 1924.

root a still more cumbersome method was employed. Thus, in finding the cube root of 100, Heron takes the two nearest cubes 64 and 125, with errors 36 and 25; by proportioning these errors to the cube roots of 64 and 125 by a process whose theoretical basis he does not fully explain he arrives at $4\frac{9}{14}$ as the approximate cube root of 100. (Method of Double False Position). Among the Egyptians and Babylonians their crude methods of false position and trial and error entailed so much labor that the results were perserved in texts and tables in much the same way as we keep tables of logarithms[1]. The Greeks did the same. We can appreciate these labors if we remember that the Egyptians, Babylonians, Greeks, and Romans had no place value of notation, and only the most meager sort of functional and operational symbolism.

If the reader will try to perform Heron's problems using only Roman numerals he will be able better to appreciate the method used in our present day text books.

II.

The first recorded instance of a method similar to our own is that given by Theon of Alexandria, the Greek astronomer (*c* 375 *A. D.*). The reader may know him better as the father of the heroine in Kingsley's novel, *Hypatia*. In his commentaries on Ptolemy he approximates the square root of irrational numbers by applying the geometrical counterpart of the formula $(a + b)^2 = a^2 + 2ab + b^2$. As the Greeks used sexagesimal fractions and had no place value notation nor any symbol for zero, Theon's evolution of digits could not be made very expeditious, and he found it useful only for irrational square roots, and he nowhere tries it for cube roots.

Quoting Euclid II, 4, "If a straight line be divided at any point, the square of the whole line is equal to the squares of both the segments together with twice the rectangle contained by the segments", he finds the square root of 4500° as follows:

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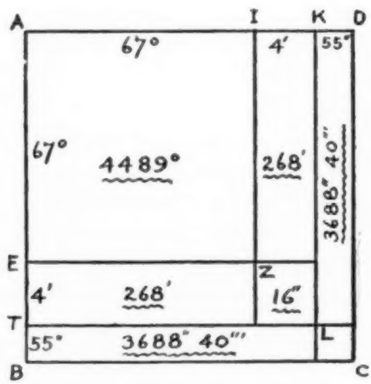
[1]Two noted examples of these are the Ahmes papyrus in Egypt and the Senkereh tablets in Babylonia. For a detailed account of various ancient methods of extracting roots of numbers see M. A. Nordgaard's *A Historical Survey of Algebraic Methods of Approximating the Roots of Numerical Higher Equations Up to the Year 1819*, (Teachers College, Columbia University, Contributions to Education, No. 123), chapters 2 and 3.

Suppose the square AC contains 4500°. The nearest square number is 4489°, and is represented by the square AZ, whose side AI is 67°. The remainder $BZDC$ contains 11°. Dividing 11° by 134° ($=2AI$) gives 4′ as the side of the second square ZL. We now have a new square AL composed of the square AZ ($=4489°$), the two rectangles TZ ($=268'$) and KZ ($=268'$), and the square ZL ($=4'\cdot 4'=16''$). The side AK of square AL is 67° 4′. The remainder $BLDC$ contains 7424″. Dividing this by twice 67° 4′ gives 55″ as the approximate value of KD. Hence the square root of 4500° is 67° 4′ 55″, approximately.



Following this demonstration he enunciates his general rule: "When we seek the square root, we first take the root of the nearest square number. We then double this and divide with it the remainder reduced to minutes and subtract the square of the quotient; then we reduce the remainder to seconds and divide by twice the degrees and minutes [of the whole quotient]. We thus obtain nearly the square root of the quadratic."

III.

Our method, however, was inherited from the Hindus and not from the Greeks. There are two reasons why the former made the extraction of roots more of a success than the latter. In the first place they had a place value notation and later on a

concept of zero as a number and a symbol for the same: the Greeks had neither a place value notation nor a zero. In the second place the Hindu mind had a natural aptitude for operating by the process of inversion. Says Aryabhata: "Multiplication becomes division, division becomes multiplication; what was gain becomes loss, what loss, gain; inversion." Aryabhata, Brahmagupta, Mahāvīra, Srīdhara, and Bhāskara first show how to find the squares and cubes of numbers; then, by inversion, they promulgate the rules for square and cube roots.

The earliest known directions for finding the square and cube root by the orderly evolution of digits and based on the place value notation, is given by Aryabhata (b. 476 *A. D.*) He did not use the characters that later developed into the Hindu-Arabic numerals, but he employed an alphabetic notation that had place value and some authorities even claim that his expression for "void" implies a knowledge of the qualities of zero.

At the time of Brahmagupta (b. 598 *A. D.*), over a century later, the Hindu numeral characters had come into use, and we find him giving definite rules for operating by and on zero as a number. He gives no detailed rule for square root, seeming to take a knowledge of that process for granted. But for the cube root he gives the same rule as Aryabhata, and his commentator Chaturveda gives a definite mode for setting up the work. The later writers Mahāvīra (*c.* 850), Srīdhara (*c* 1020), and Bhāskara (*c.* 1140) give the same rules in more complete form. For the language of the scientific classics of the Hindus is very cryptic, since it was their custom to write their treatises in verse. The interpretation and execution of their epigrammatic rules was mostly left to their commentators.

Aryabhata, and the Hindu writers after him, divided the radicand into periods of two and three digits for square and cube root, respectively, beginning at the right. In the former the first, third, etc. digits were called quadratics; in the latter the first, fourth, etc., were called cubics; the remaining digits were called non-quadratics or non-cubes. The later commentators designated the quadratics and cubics by vertical strokes and the other digits by horizontal bars, thus: $\overset{-}{9}\overset{|}{0}\overset{-}{2}\overset{|}{5}$, $\overset{|}{1}\overset{-}{7}\overset{-}{2}\overset{|}{8}$.

They aimed to make the operation automatic. For the calculation was performed on a board twelve inches by ten, covered

with clay dust, and the smallness of space necessitated a definite order so that the figures could be erased as the operation progressed. In this connection Chaturveda specifically enjoins that a separate line be reserved for the figures of the root as they evolve. Since mechanical limitations prevented them from preserving on slate or paper the results of the operations as they went along, they used an order different from ours. Where we use the device of the "complete divisor" and get along with one subtraction for each digit of the root, they used only the "trial divisor" and subtracted each term as they went along. That is, where we use the expression $a^2 + (2a + b)b$ and $a^3 + (3a^2 + 3ab + b^2)b$, they invariably exhausted term by term the expressions $a^2 + 2ab + b^2$ and $a^3 + 3a^2b + 3ab^2 + b^3$. (Notice the illustration from Bhāskara.)

The following is Aryabhata's rule for square root given in the 4th book of the Aryabhatizam: "Always divide the non-quadratic by double the root of the quadratic [preceding], after having subtracted from that quadratic the square of the root: the quotient is the root of the next figure removed one place." This rule is very similar to that of Theon. But the fuller forms of the rule given by Mahāvīra, Srīdhara, and Bhāskara describe how the work was set up: every digit found was *doubled* before it was placed in the "line of the roots", so that the part of the root already found would at every stage be the *trial divisor* in finding the next digit of the root. Rule 36 of the *Ganita-Sara* of Mahāvīra says:

"From the [number represented by the figure up to the] last odd place [of notation counted from the right], subtract the [highest possible] square number; then multiply the root [of this number] by two, and divide with this [product the number represented by taking into position the figure belonging to] the [next] even place; and then the square of the quotient [so obtained] is to be subtracted from the [number represented by taking into position the figure belonging to the next] odd place. [If it is so continued till the end], the half of the [last] doubled quantity [comes to be] the resulting square root."

Srīdhara, in the *Trisátika* (c. 1020) and Bhāskara, in *Lilawati* (c. 1140) give the same rule. We take an illustration from the latter. The marginal explanations are ours.

$a^2 + 2ab + b^2$	$\overset{\mid\,-\,\mid\,-\,\mid}{8\,8\,2\,0\,9}$	Root line	
a^2	4	$2 \times 2 \ldots 4$	$2a$
$2a\ )2ab + b^2$	$4\ )\ 4\,\overline{8}$	$9 \times 2 \ldots 18$	$2b$
$2ab$	$3\,6$	58	$2(a + b) = 2a,$
b^2	$1\,2\,2$		
b^2	$8\,1$	$7 \times 2 \ldots 14$	$2b,$
$2a,\ )2a, b, + b,^2$	$58\ )\ 4\,1\,\overline{0}$	$2\ )594$	$2\,(a, + b,)$
$2a, b,$	$4\,0\,6$	$297,$	$a, + b,$
$b,^2$	$4\,9$	the root	
$b,^2$	$4\,9$		

To peoples not using boards covered with clay dust the Hindu device of doubling the digits of the root in the root line possessed no advantage and we do not find it practiced among the Europeans. The Hindus themselves had no similar arrangement for cube root.

For extracting the cube root Aryabhata gives this direction, also in cryptic verse: "Divide the second non-cubic digit by three times the square of the root of the cubic [preceding]; its square, multiplied by three times the first [number found], must be subtracted from the first [non-cubic], and the cube from the entire cubic [digit]." It must be understood, of course, that $3a^2b$ is to be subtracted first. The same rule is repeated by Brahmagupta, Mahāvīra, Srīdhara, and Bhāskara. The chief service of Bhāskara was to amplify the language and emphasize the arrangement of the work. We quote him, as translated by Taylor[1]: "The first place on the right is called *ghana* or cube; the two next places *aghana* or not-cube. Subtract the cube contained in the final period from the said period; put down the root *of the cube* in a separate line, and after multiplying its square by three, divide the antecedent figure by the result, and write down the quotient in the separate line: Then multiply the square of the quotient by the preceding number *in that line* and by three, and after subtracting the product from the next antecedent figure cube the said quotient, and subtract the result from the next antecedent figure. Thus repeat the process through all the figures. The separate line contains the Cube Root."

[1] Lilawati, editor Taylor, Bombay, 1816, p. 20.

Notice the elegant way in which Bhāskara's commentator extracts the cube root of 1953125. Alongside of it is placed our modern process. Incidentally let the reader notice our modern device of "complete divisor" to take the place of the Hindu term-by-term exhaustion, and let him reflect whether our way is really an improvement.

			Root line
$a^3 + 3a^2b + 3ab^2 + b^3$		1 9 5 3 1 2 5	
a^2		1	1
	3	9	2
$3a^2b$		6	
		3 5	
$3ab^2$		1 2	
		2 3 3	
b^3		8	
	432	2 2 5 1	5
$3a_1{}^2b_1$		2 1 6 0	
		9 1 2	
$3a_1b_1{}^2$		9 0 0	
		1 2 5	
$b_1{}^3$		1 2 5	

		1,953,125 (125
a^3		1
$3a^2$	300	953
$3ab$	60	
b^2	4	
$(3a^2 + 3ab + b^2)b$	364	728
$3a_1{}^2$	43200	225125
$3a_1b_1$	1800	
$b_1{}^2$	25	
$(3a_1{}^2 + 3a_1b_1 + b_1{}^2)b_1$	45025	225125

The Hindu method of extracting roots, and especially Bhaskara's presentation of it, was adopted by the Arabs and by them

disseminated through all the parts of the civilized world. It was called the "Hindu method." As far as the author has been able to investigate, they did not use this method for roots higher than the third. Omar Khayyam, the Persian poet and mathematician, who flourished shortly before Bhāskara, claims that he extracted fourth, fifth, and sixth roots arithmetically and that his method was based on the arithmetical portion of Euclid's *Elements,* and that such operations had not been effected before.[1] This seems to indicate that he extended the Hindu method to these higher roots. Unfortunately the work which he refers to is lost. The plan of putting down the double root in the square root line was soon abandoned for the more direct and natural method used in finding cube root, and which was possibly used by Aryabhata also for square root.

IV.

The Hindu method of extracting arithmetical roots was made known to European scholars by Leonardo of Pisa in his *Liber Abaci* (1202); in this work he gave an account of the algebra of the Arabs and introduced and explained to the Europeans the advantage of the Hindu-Arabic numerals. In extracting roots he arranged his work so that the root (quotient) and the multipliers were placed beneath the radicand and the several remainders above the radicand, as shown by the following two illustrations from *Liber Abaci,* in which the square root of 8754 and the cube root of 56789 are to be found.[2]

6	11	105
87	5	4
	9	3
	9	3

1	
2	
5	9
8	9
3	1
1	44
29	127
56	789
	38
	27
	192
	64

[1] Cantor, I (1894), p. 732; *"L' Algebre d'Omar Alkayyami"* (ed. Woepcke) 13.

[2] *Scritti di Leonardo,* I, Boncompagni, Rome, 1857, p. 354; p. 382.

In the first, the root is 93 and the residue 105. The first three steps, explained rhetorically by Leonardo, are as follows:

	6	6 11
87 54	97 54	87 54
9	9	93
	9	93

In the second, where the root is 38 and the residue 1917, the first two steps would be if separated:

	29
56 789	56 789
3	3
	27

Leonardo's work, admirably clear and scholarly, was too advanced for the general European learning of the time, and it remained for the more elementary treatises of Sacrobosco (died 1256), Alexander de Villedieu and Petrus de Dacia, in the latter part of the same century, to make the algorithmic processes a thing of general knowledge. Sacrobosco's arithmetic became the textbook of Europe for two hundred years, and his rules for finding arithmetical roots by the Hindu method were used up to the time of Peurbach in the fifteenth century.

Though the physical limitations of the Hindu sandboard computations no longer existed, their plan of work still remained in use; and their ideal of compactness persisted when it had ceased to have any virtue. This was changed by the noted astronomer, George Peurbach (1423-61). When he found it necessary to check Sacrobosco's tables for his astronomical work, he instituted an arrangement whereby the eye could clearly see what had been done at any stage of the work and what remained to be done. It is this new idea which distinguishes his *schema* and the later modifications of it from the arrangement of Sacrobosco.

To do this he brings in three new devices: (1) he separates the radicand into periods by means of dots; (2) he places the root on the right of the radicand; (3) he crosses out the used digits where the Hindus had erased them and Leonardo had left them intact. Peurbach's illustration follows:

2	$\cancel{2}\cancel{3}\cancel{5}$	$\cancel{2}\cancel{3}\cancel{5}$
$\dot{\cancel{6}}6\dot{0}4\dot{9}(2$	$\dot{\cancel{6}}\cancel{6}\dot{\cancel{0}}4\dot{9}(26$	$\dot{\cancel{6}}\cancel{6}\dot{\cancel{0}}\cancel{4}\dot{\cancel{9}}(257$
4	4	50
	225	3549

We note that Peurbach differed from the medieval writers (1) in his using, though not writing out, the "complete divisor," and (2) in his standing out the trial divisor and the subtrahend before the eye.

Compare Peurbach's *schema* with that used in modern arithmetics:

$$\begin{array}{rl} & 6,\ 60,\ 49\ (\ 257 \\ & 4 \\ \cline{2-2} 45\ ) & 2\ 60 \\ & 2\ 25 \\ \cline{2-2} 507\ ) & \quad 35\ 49 \\ & \quad 35\ 49 \\ \cline{2-2} \end{array}$$

Peurbach's prestige as an astronomer and a mathemetician caused his *schema* to be adopted in all parts of Europe. A knowledge of this was gained through his lectures to the students of the University of Vienna (*c.* 1454) and through the many editions of his posthumously published arithmetic. Nicholas Chuquet, a brilliant French mathematician, adopted this method in his *Triparty* (1484) and modified it by setting the root *under* the radicand, yet separated from the rest of the work by cross lines; underneath the root, and to the left of each new figure of the root, he sets the double of the root already found so that the complete divisor stands out to the eye. Thus, in Peurbach's example, the process would be:

$$\begin{array}{c|cc|cc} & \cancel{3} & & & \\ \cancel{2} & \cancel{6} & \cancel{5} & & \\ \cancel{6} & \cancel{6} & \cancel{0} & \cancel{4} & \cancel{9} \\ \hline 2 & & 5 & & 7 \\ \hline & \cancel{4} & \cancel{5} & \cancel{0} & \end{array}$$

Chuquet's *Triparty* was not printed till 1880; but it circulated in manuscript and exerted much influence. In the little book of Pedro Ciruelo Sanchez, a Spaniard residing in Paris, published in 1495, Chuquet's scheme is used; likewise in the arithmetics of La Roche (1520) and Cardan (1539).

In this scheme, however, the subtrahends do not stand out as plain as in Peurbach. Gema Frisius (1540) effects an ar-

rangement whereby the subtrahend stands out and where—for the first time, we believe—the new digit of the root can be added to twice the root already found, all in one line. Again we use Peurbach's example to illustrate:

```
2 3 5
6 6 0 4 9 ( 2 5 7
·   ·   ·
4
  4 5
---------
2 2 5
    5 0 7
---------
  3 5 4 9
```

The cube root process employed substantially the same arrangement. Leonardo of Pisa put the root and the divisors underneath the radicand and the remainders above, as in this illustration, where the cube root of 56,789 is found to be 38 and the residue 1917.

```
  1
  2
  5 9
  8 9
  3   1
1   4 4
2 9 1 2 7
5 6 7 8 9
      3 8
    2 7
    1 9 2
      6 4
```

Peurbach gives no illustration of how to find the cube root; but following his minute directions for cube root and his mode of setting up the work in square root, we should find the cube root of 94818816, say, as follows:

	3	
	3 0 6 9 3 2	$a \quad b$
$a^3 + 3a^2b + 3ab^2 + b^3$:	9 4 8 1 8 8 1 6 (	$\overbrace{4\ 5}^{}\ 6$
$3a$:	1 2	$a_1 \quad b_1$
$a + b$:	4 5	
$3a^2$:	4 8	
$3ab$:	6 0	
$3a^2 + 3ab$:	5 4 0	
b:	5	
$(3a^2 + 3ab)b$:	2 7 0 0	
b^3:	1 2 5	
	2 7 1 2 5	
$3b$:	1 5	
$3a$:	1 2	
$3(a + b) = 3a_1$:	1 3 5	
$a_1 + b_1$:	4 5 6	
$3a_1^2 + 3a_1b_1$:	6 1 5 6 0	
b_1:	6	
$(3a_1^2 + 3a_1b_1)b_1$:	3 6 9 3 6 0	
b_1^3:	2 1 6	

Practically the only difference between Peurbach's process and our present one is his rather awkward way of getting the trial divisor; he uses also the only partially complete divisor: $3a^2 + 3ab$ where we use $3a^2 + 3ab + b^2$. La Roche (1520), Chr. Rudolff (1525), Apian (1532), Cardan (1539), Gemma Frisius (1540), Stifel (1544), Ramus (1555, 1569), Menher (1565), and Stevin (1584), all use Peurbach's *schema* as a basis, though varying in the details of arrangement.

The Hindu and Renaissance writers used the place value element more extensively than we do at present: we find it advantageous to treat the completion of the divisor in the cube root differently from that used in square root. That plan seems first to have been followed by Stifel and Stevin. In the accompanying comparative solutions note Gemma Frisius's use of 240 and 300 and 125 where Stifel uses 24000 and 3000 and 125.

Gemma Frisius (1540)

3
~~30~~693
~~94818~~816 (456
12
48
240
300
125
27125
135
6075
36450
4860
216
3693816

Stifel (1544)

16—300— 5
4— 30—25
125

2025—300— 6
45— 30—36
216

24000
3000
125
27125

3645000
48600
216
3693816

3
~~30~~693
~~94818~~816 (456

The plan of exhausting term by term continued to be used freely. It was not till the latter half of the nineteenth century that exhausting by means of completing the divisor came to be used exclusively. It is a questionable improvement.

Strangely enough the Hindu scholars did not use their method in the field where it has actually rendered the greatest service, namely in extracting the roots of irrational quantities. Their aim was only to facilitate finding roots of *large numbers.* For irrational numbers they used methods of false position and Theon's formula $\sqrt{a^2+b}=a+\frac{b}{2a}$. But whereas Theon, who found this method of advantage only for the fractional part of the root, evolved the digits showing minutes, seconds, thirds, etc., in the sexagesimal system, the Hindu writers did not evolve the digits of tenths, hundredths, etc. For decimal fractions were yet in their infancy, and there was no decimal point notation.

It remained for Christian Europe to extend the Hindu method into the domain of decimal fractions. Theon's rule was used extensively all through the Middle Ages. Al-Karchi (*c.* 1020) used this modification of it: $\sqrt{a^2+b}=\mathrm{a}+\frac{b}{2a+1}$;

and Leonard of Pisa (1202) frequently employed it with a subtractive correction in the form $\sqrt{a^2+b} = a + \frac{b}{2a} - \frac{\left(\frac{b}{2a}\right)^2}{2\left(a+\frac{b}{2a}\right)}$. There was no corresponding formula for the cube root until Leonardo invented the approximation formula $\sqrt[3]{a^3+b} = a + \frac{b}{3a(a+1)+1}$. Leonardo employed the Hindu method for the integral part of the root and the formulas for the fractional part. His solution of the example already quoted is as follows: $\sqrt{8754} = \sqrt{93^2+105} = 93 + \frac{105}{2\cdot 93}$, omitting the correction. An even more effective move was Leonardo's adaptation of the formula $\sqrt{A} = \frac{1}{a}\sqrt{Aa^2}$, which was used by John Hispalensis and others in the Middle Ages for approximation in sexagesimal fractions. By letting a be 10, 100, 1000, etc., Leonardo could get any desired degree of accuracy. He combines this witb Theon's formula in one of his illustrations: $\sqrt{7234} = \frac{1}{100}\sqrt{72340000} = \frac{1}{100}\cdot 8505\frac{1}{4} = 85\frac{1}{20}\,\frac{1}{400}$. The process worked equally well for the higher roots. It was the property of the Middle Ages from then on and gained in favor coincident with the ascendency of decimal fractions over sexagesimal fractions.

It only needed a mathematician of power and prestige to give scientific directions and a workable *schema,* such as did Bhāskara and Peurbach in their days, for this method to become the standard method. Such a man came in the person of Cardan. He gives this rule for irrational square roots: "Add to the number whose square root is to be found as many 00's as you desire places of approximate accuracy. When 00 is added, you get a result correct to one-tenth; when 0000 is added a result correct to one-hundredth is obtained; adding 000000 gives a result accurate to one-thousandth . . . , and so continuously to half the number of zeros." He gives a similar rule for cube root. To get the square root of 17 he

evaluates $\sqrt{1700000000} = 41231$; hence $\sqrt{17} = 4\frac{1231}{10000}$. Similarly $\sqrt[3]{17000000000000}$ gives 25712; hence $\sqrt[3]{17} = 2\frac{5712}{10000}$ (See Chapter 23 in Cardan's Arithmetic).

Thus, even without a decimal point, the world had learned to harness the Hindu method to approximate irrational roots. With the introduction of the decimal point notation, for which Steven's *La Disme* (1585) is largely to be credited, the method became as nearly perfected and automatic as we can hope for any approximation method to become.

Summary

1. Our present method of extracting roots by the evolution of digits depends upon the inverse operation of the binomial expansion and a notation employing place value and zero.

2. Theon of Alexandria (*c.* 375 A. D.) approximated irrational square roots in sexagesimal fractions by evolving the digits of the successive orders. This method entailed considerable work, for his notation had no zero and no place value.

3. Aryabhata (b. 476) enunciated rules for evolution dependent on an alphabetic notation with place values. He may even have employed the zero concept.

4. Brahmagupta (b. 598), Mahāvira (*c.* 850), Srīdhara (*c.* 1020), and Bhāskara (*c.* 1040) stated the same rules based upon the Hindu-Arabic numerals, including the zero.

5. The Hindu method was adopted by the Arabs, and by them transmitted to the Europeans, who learned of it through the writings of Leonardo of Pisa (1202), Sacrobosco (d. 1256), Alexander de Villedieu, and Petrus de Dacia in the latter part of the fifteenth century.

6. Georg Peurbach (c. 1454) worked out a *schema* whereby is indicated the stage of progress of the operation and the steps that remain to be done. This was modified, and sometimes improved, in the arithmetics of Chuquet (1484), Pedro Sanchez Ciruelo (1544), Stevin (1584), and others.

7. The Hindus used their method for finding the roots of large numbers. Their method of evolution was extended to the domain of decimal fractions through the work of John Hispalensis, Leonardo of Pisa (1202), Cardan (1539), and Stevin (1585).

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NEWS AND NOTES

THE Association of Teachers of Mathematics in Southern Massachusetts held its winter meeting in Fall River, February 9, 1924. The program included:

1. The Imaginary in Geometry, Prof. William C. Graustein, Harvard University.

2. Standardized Test in Plane Geometry, Miss Vera Sanford, The Lincoln School.

3. The Making of a Comprehensive Examination for the College Examination Board, Mr. Edmond D. Searls, New Bedford High School.

4. The Course of Freshman Mathematics at Brown, Professor R. W. Burgess, Brown University.

The officers of the Association are: President, Mr. James L. Cummings, Fall River; Vice-President, Miss Mary Carroll, New Bedford; Sec.-Treas., Miss Margaret Macdonald, Fall River.

THE Southeastern Section of the Mathematics Association of America announces the following program to be held at the University of Georgia on March seventh and eighth, 1924:

Friday, March 7—

8:00 p. m. Special Dinner in honor of Professor Slaught.

9:00 p. m. Address by Professor H. E. Slaught, University of Chicago. "The Association, Its Ideals, Accomplishments and Prospects."

10:00 p. m. Social Hour.

Saturday, March 8—

10:00 a. m. "Imaginary and Infinite Elements in Undergraduate and High School Teaching." Professor Tomlinson Fort, University of Alabama. 20 minutes.

"Teaching of Mathematics at the University of Georgia." Professor R. P. Stephens, University of Georgia. 20 minutes.

"Mathematics and the Other Sciences." Professor H. E. Slaught, University of Chicago. 35 minutes.

"Some Contacts of Projective Geometry with Elementary Mathematics." Professor J. W. Lasley, Jr., University of North Carolina. 20 minutes.

"The Cultural Value of Mathematics." Professor W. W. Rankin, Jr., Agnes Scott, Decatur, Georgia. 15 minutes.

Discussion—"What Can the Association Do to Aid Individual Departments." Led by Professor Fort.

In referring to the experiments for testing the Einstein relativity theory at the University of Chicago, President Ernest DeWitt Burton at the recent convocation quoted the following statement by Dean Henry G. Gale of the experiment which he and Professor Albert A. Michelson, head of the Department of Physics, are now making:

"Messrs. Michelson and Gale have completed preliminary tests on their ether drift experiment which was designed to ascertain whether or not a beam of light traveling in a closed circuit on the earth's surface experiences a drag as a result of the earth's rotation. The preliminary experiments have shown that the difficulties of the measuremnt can be surmounted, and work is under way in preparation for the final experiment which will be carried out in the spring."

President Burton added that the other men whose researches Dean Gale reported are over thirty in number.

The Association of Teachers of Mathematics in New England held its mid-winter meeting at Worcester, Massachusetts, on Saturday, March 15, 1924. The program included: Address of Welcome, President Ira N. Hollis, Worcester Polytechnic Institute; "Four Years of Mathematics for Commercial Pupils," Frederick K. Hussey, Newton Technical High School; "Linkages," Professor Raymond K. Morley, Worcester Polytechnic Institute; "Dynamic Symmetry—A Long Lost Contact Between Geometry and Art," Miss Helen E. Cleaves, Manual Arts Department, Boston Normal School; "The Rhind Mathematical Papyrus," Chancellor A. B. Chase, Brown University.

Council for 1924 consists of: A. Harry Wheeler, Professor Lennie P. Copeland, Harry D. Gaylord, Harold B. Garland, Professor J. W. Young, Charles H. Mergendahl, Harry C. Barber, Miss Olive A. Kee, Professor George D. Birkhoff and Miss Annie W. Doughty.

Courses announced for the summer of 1924 at the University of Chicago are: By Professor G. A. Bliss, Functions of a Real Variable, 4 hours; Thesis Work in Analysis. By Professor L. E. Dickson, Theory of Numbers I, 4 hours; Thesis Work in Number Theory. By Professor H. E. Slaught, Elliptic Integrals, 4 hours; Differential Equations, 4 hours. By Professor M. Frechet,

Theory of Abstract Sets, 4 hours; Theory of Probability, 4 hours. By Professor E. T. Bell, General Theory of Numbers, 4 hours; Theory of Equations, 4 hours. By Professor F. R. Moulton, Functions of Infinitely Many Variables, 4 hours; Analytic Mechanics II, 4 hours. By Associate Professor J. W. A. Young, Differential Calculus, 5 hours; College Algebra, 5 hours. By Assistant Professor E. P. Lane, Synthetic Projective Geometry, 4 hours; Plane Analytic Geometry, 5 hours. By Dr. Mayme I. Logsdon, Introduction to Higher Algebra, 4 hours; Integral Calculus, 5 hours.

ARTICLES OF INTEREST

Mathematicians and Music. By R. C. ARCHIBALD, Brown University, *American Mathematical Monthly*, January 1924, Pp. 1-25.

Presidential Address delivered before the Mathematican Association of America, September 6, 1923. With the numerous footnotes added this article is a valuable reference.

The Teachers' Responsibility For Our Educational Integrity. By HENRY S. PRITCHETT, President of the Carnegie Foundation, School and Society, Pp. 113, February 2, 1924.

An address delivered before the Association of American Colleges. It is well worth reading by every teacher.

The Humanities Versus the Utilities. By FLORENCE MAY BENNETT, Walla Walla, Washington, and

Some of My Worst Teaching Mistakes. By a School Principal, both in *Education*, February, 1924. ALFRED DAVIS.

NEW BOOKS

Elements of the Theory of Infinite Processes. By Lloyd L. Smail, University of Oregon. McGraw-Hill Book Company, New York. Pp. 339.

Essentials of Algebra. By David Eugene Smith and William D. Reeve. Ginn and Company.

Modern Mathematics, Seventh School Year. Modern Mathematics, Eighth School Year. Modern Algebra, Ninth School Year. By Raleigh Schorling and John R. Clark. World Book Company.

Junior High School Education. By Calvin O. Davis, World Book Company, Yonkers, N. Y. 1924. Pp. 444.

This book should be examined by every teacher of grades seven, eight and nine. It states the purposes of junior high school education, describes curricula and organization of outstanding junior high schools, and discusses the methods and materials of the different subjects of instruction.

Three recent numbers of the Ostwald Klassiker der exakten Wissenschaften. Leipzig, Akademische Verlagsgesellschaft.

In the last two years three numbers of these classics in mathematics and science have come to the reviewer's table. These are (1) The work of Bartholinus on the peculiar refraction which characterizes Iceland Spar a work which appeared in Copenhagen in 1669, and which is now (1922) translated from the Latin into German by Karl Mieleitner (35 pages); (2) The work of M. von Laue and his collaborators Friedrich, Knipping, and Tank, on the interferance of Rontgen rays, first prepared in 1912-1914 and now edited by F. Rinne and E. Schiebold (111 pages & plates); (3) Fermat's notes on plane and solid loci, made about 1636, and now (1923) translated from the Latin by H. Wieleitner (22 pages).

Erasmus Bartholinus (1625-1698) was a member of a remarkable family, one of the six sons of Caspar Berthelsen (1585-1629), three of whom ranked high as scientific investigators. His

Experimenta crystalli islandici was the first work to describe the double refraction of this kind of crystal, and it is a help to historians of science to have the memoir in a modern language and at a nominal cost.

The work on Rontgen rays is not as yet a classic in the sense of that of Bartholinus, but its appearance in this form will doubtless be welcomed by all physicists.

To the mathematician, however, it is the little essay of Pierre de Fermat (1608-1665) that will be most welcome, since he will find here the essence of his discoveries in analytic geometry. The essay is also to be found in the edition of Fermat's works prepared by P. Tannery and Ch. Henry in 1891; but to have it by itself, inexpensively printed, will be a great help to scholars who are interested in historical matters and who have some command of German.

David Eugene Smith.

Relativity—A Systematic Treatment of Einstein's Theory. Longmans, Green & Co. P. 389. Price, $6.

This book puts in the hands of our college students the mathematical and physical contributions of Einstein, Weyl, and Eddington, together with others. This text is a happy balance between the mathematical and physical points of view. It is for the science undergraduate that this book is written and in difficulty lies between the liberal supply of popular works and the advanced treatises on the subject.

After an introduction in which he explains in general terms the "why" and "how" of the theory he proceeds to a detailed account of the Restricted and General Theory and World Geometry.

Throughout the entire book he emphasizes clearly that:

(a) Each new contribution has been a generalization of the proceeding.

(b) That these successive generalizations have been accomplished "by the modification of its mathematical method but not its point of view."

(c) The unalterable feature has in all cases been the invariance of the mathematical forms of the equations of physics.

Besides developing carefully each mathematical and physical concept he stresses the correspondence between the two fields and the translation from one to the other.

The use of the book by the science undergraduate in an American college or university is questioned. In all probability it would be mostly confined to that increasing group of undergraduates termed, "students reading for honors." That the author has some doubt as to the use of the entire book by the English science undergraduate is shown by the quotation: "Doubtless the reading of these two parts [Restricted and General Relativity] will involve as much time and thought as even the most industrious student can spare from the demands of the other branches of the physical and mathematical sciences in his undergraduate years. But no book on relativity published at present would be complete without some account of the very interesting developments which have taken place since 1917. Not only for the sake of post-graduate reading, but in order to appeal to a wider circle of readers, a fairly complete account is given in Part III of the cosmological speculations of Einstein and de Sitter and the attempts of Weyl, Eddington and Einstein to derive a mathematical theory of the electromagnetic field (as well as of the gravitational) from the treatment of the metric field of spacetime."

This book deserves the consideration of all those interested in this subject.

G. R. Mirick.

The Lincoln School.

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